Applications

Student Name	
Department	
Section No.	

Exercises 1

Prove that

$$\boxed{\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1}$$

Exercises 2

If $z = x + jy$, find the equation of the locus $\arg(z^2) = \frac{\pi}{4}$

Exercises 3

1-Expand $\sin 4\theta$ in powers of $\sin\theta$ and $\cos\theta$.

2- Express $\cos^4\theta$ in terms of cosines of multiples of θ.

3- If $z = x + jy$, find the equations of the two loci defined by

(a) $|z-4| = 3$ (b) $\arg(z+2) = \dfrac{\pi}{6}$

Exercises 4

Show that $u(x, y) = x^3y - y^3x$ is an harmonic function and find the function $v(x, y)$ that ensures that $f(z) = u(x, u) + jv(x, y)$ is analytic. That is, find the function $v(x, y)$ that is conjugate to $u(x, y)$.

Exercises 5 (Harmonic functions)

Are the following functions harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

1. $u = e^{-x} \sin 2y$
2. $u = xy$
3. $v = xy$
4. $v = -y/(x^2 + y^2)$
5. $u = \ln |z|$
6. $v = \ln |z|$
7. $u = x^3 - 3xy^2$
8. $u = 1/(x^2 + y^2)$
9. $v = (x^2 - y^2)^2$

Exercises 6

Determine a, b, c such that the given functions are harmonic and find a harmonic conjugate.

1- $U = ax^2 + y^2$

2. $u = e^{3x} \cos ay$

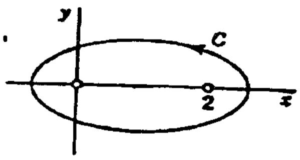
3. $u = \sin x \cosh cy$

4. $u = ax^3 + 5y^3$

Exercises 7

Evaluate

$\oint_C \frac{7z-6}{z^2-2z}\,dz$, C as shown



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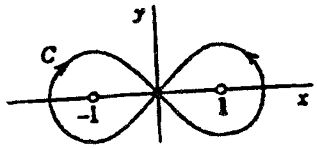
xercises 8

valuate

$$\oint_C \frac{dz}{z^2 - 1}\ ,\ C \text{ as shown}$$



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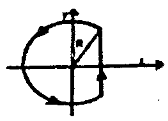
Exercises 9

(a) **Evaluate** $\oint_C \frac{z}{(z-1)(z+2i)}\,dz$ **around** $C: |z| = 4$.

(b) Using Bromwich contour



To find inverse Laplace transform of

$$F(s) = \frac{1}{(s-1)(s-2)}$$

xercises 10

xpand $\dfrac{e^{3z}}{(z-2)^4}$ in a Laurent series about the point $z = 2$ anc

etermine the nature of the singularity at $z = 2$.

Exercises 11

Find the Laurent series about the point indicated of each of the following.

(a) $\frac{1}{z}\sin\left(\frac{1}{z}\right)$ about the point $z = 0$

(b) $\frac{1}{2z-3}$ about the point $z = 3/2$

(c) $\frac{z}{(z-2)(z-3)}$ about the point $z = 3$.

Find the Laurent series of $\frac{z-1}{(z+2)(z+5)}$ that is valid for

(a) $2 < |z| < 5$

(b) $|z| > 5$

(c) $|z| < 2$.

ercises 12

valuate $\int_0^{2\pi} \frac{1}{4\cos\theta - 5}\, d\theta.$

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Exercises 13

Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^4}\,dx$.

Exercises 14

1 Map the following points in the z-plane onto the w-plane under the transformation $w = f(z)$.
(a) $z = 3 + j2$; $w = 2z - j6$
(b) $z = -2 + j$; $w = 4 + jz$
(c) $z = j(1 - j)$; $w = (2 + j)z - 1$
(d) $z = j - 2$; $w = (1 - j)(z + 3)$.

2 Map the straight line joining A $(2 - j)$ and B $(4 - j3)$ in the z-plane onto the w-plane using the transformation $w = (1 + j2)z + 1 - j3$. State the magnification, rotation and translation involved.

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Exercises 15

Evaluate the integrations using Gamma and beta function

(i) $\int_0^\infty \sqrt{x}\; e^{-x^3}\, dx$ (ii) $\int_0^1 x^m (\ln x)^n\, dx$

(iii) $\int_0^1 \sqrt{\frac{1}{x} - 1}\; dx$

ercises 16

aluate the integrations using Gamma and Beta functions

$\int_0^{\infty} (1+\sqrt{x})^2 \; e^{-x} \; dx$ (ii) $\int_{-\infty}^{\infty} e^{(3x-e^x)} \; dx$

i) $\int_0^1 (x^3+\sqrt{x})^2 \; \sqrt{1-x} \; dx$

$\int_2^5 (x-2)^3 \; (5-x)^{\frac{1}{3}} dx$

Exercises 17

(a)Prove that

(i) $e^{ix\sin\theta} = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)\cos 2n\theta + 2i\sum_{n=0}^{\infty} J_{2n+1}(x)\sin(2n+1)\theta$

(ii) $1 = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)$

(iii) $x = 2\sum_{n=0}^{\infty}(2n+1)\, J_{2n+1}(x)$

prove that $+2J_4\cos(4\theta)=J_0+2J_2\cos(2\theta$ cos(xsin

$)+\dots\theta)$ $+2J_6\cos(6\theta)+2J_4\cos(4\theta)=J_0+2J_2\cos(2\theta$ sin(xsin

$)+\dots\theta)$ $+2J_5\sin(5\theta)+2J_3\sin(3\theta)=2J_1\sin(\theta$

Let sets A, B and C be fuzzy sets defined on real numbers by membership functions

$$\mathfrak{A}_A(x)=\frac{x}{x+1}\quad,\mathfrak{A}_B(x)=\frac{1}{x^2+10}\,,\mathfrak{A}_C(x)=\frac{1}{10^x}$$

Determine mathematical membership functions graphs of the followings

$a)\ A\cup B\ ,\ B\cap C\ ,\quad b)\ A\cup B\cup C\ ,\ A\cap B\cap C$

$c)\ A\cap C\ \ ,\ B\cup C\quad d)\ A\cap B\ ,A\cup B$

Show the two fuzzy sets satisfy the De Morgan s Law ,

$$\mathfrak{A}_A = \frac{1}{1+(x-10)} \quad , \quad \mathfrak{A}_B(x) = \frac{1}{1+x^2}$$

Evaluate $\oint_c \frac{e^z}{z^2+1}dz$, $\oint_c \frac{\cos z \, dz}{z^2(z+2)}$, $\oint_c \frac{dz}{z^2(z+4)}$
where C is the circle $|z-1|=2$

Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2$ is a harmonic function and find the corresponding analytic function $f(z) = u + iv$

If in the function $f(z)=u+iv$, we take z in polar form, namely $z=re^{i\theta}=r(\cos\theta+i\sin\theta)$

Show that the Cauchy - Riemann equations become

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}\ ,\ \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Example

Show that $f(z) = u(r,\theta) + iv(r,\theta)$ is analutic, then

$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$

Solution

$$u_r = \frac{1}{r} v_\theta \qquad\qquad v_r = -\frac{1}{r} u_\theta \quad \text{(C-R)}$$

iff. w.r.to r $\quad u_r = \frac{1}{r} v_\theta$

$$u_{rr} = r^{-1} v_{\theta r} - r^{-2} v_\theta$$

$$r^2 u_{rr} = r v_{\theta r} - v_\theta$$

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Form

$$u_\theta = -r v_r \qquad\qquad \text{diff - w.r.to } \theta$$

$$u_{\theta\theta} = -r v_{r\theta}$$

From (1),(2)

$$r^2 u_{rr} + r u_r + u_{\theta\theta} = r v_{\theta r} - v_\theta + v_\theta - r v_{r\theta} - r v_{r\theta}$$

$$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$$

Example

If f(z) = u(x,y) + iv(x,y) is analytic in a region R, prove that the one parameter families of cerues $U(x,y) = C1$ and $v(x,y) = C2$ are Orthogonal families.

Solution :

$u(x, y) = c_1$ **then** $du = u_x dx + u_y dy = 0$

$$\therefore \frac{dy}{dx} = -\frac{u_x}{u_y} \qquad (1)$$

Also $V(x, y) = c_2$ **then** $dv = v_x dx + v_y dy = 0$

$$\frac{dy}{dx} = -\frac{v_x}{v_y} \qquad (2)$$

By Cachy – **Rieman equations, we have the product of the** slopes at a fixed point

$$\left(-\frac{U_x}{U_y}\right)\left(-\frac{V_x}{V_y}\right) = -1$$

So that any two **members of the respective families** are Orthogonal